Mathematics in Motion – Model Rocket Velocity

Learning Objective	In this lesson the student will use mathematics to determine the velocity of a rocket. By the end of this lesson the student should have an appreciation for the real world use of mathematics as it relates to model rocketry.
Grade Level	9 – 12

How fast does a model rocket go? This may be the second most asked question after witnessing a model rocket launch. Unlike a cannon ball shot from a cannon the velocity of the model rocket actually increases after it leaves the launch pad. In this lesson we will determine the maximum velocity of a model rocket.

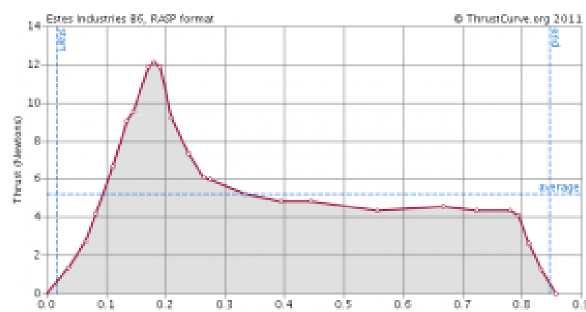

Figure 1 – Thrust Curve of an Estes B6 motor
image: ThrustCurve.com

Using Data from ThrustCurve.org

There is an organization that collects data on the performance of rocket motors called ThrustCurve.org. Figure 1 is an image taken from the ThrustCurve.org website showing the thrust curve of an Estes B6 type motor (it is important to note that the manufacturer must be taken into account as the thrust curves differ depending on the manufacturer).

It is not necessary to take the delay, or the last number in a motor classification into account when we are looking for the thrust curve of our motor. We are only concerned with the thrust phase. Thus if you search for the thrust curve for a B6-4 motor it would be the same as the thrust curve for a B6-6 motor from the same manufacturer.

The first value we will observe from the thrust curve is the Average Thrust. For this motor the Average Thrust is 5.2 Newtons. Of note here is the rounding up of this value in the motor classification (B6).

The second value to observe is the burn time. As we can see from the graph it is 0.85 seconds.

How fast should my model rocket go?

We may determine the velocity using the following formula:

$$velocity = \left(\frac{Force(Newtons)}{Weight_{average}(Newtons)} - 1\right) \times acceleration_{gravity}\frac{m}{s2} \times time_{burn}\frac{m}{s}$$

You may notice that the unit newtons is used for both the Force of the motor and the Weight of the rocket. We can measure the mass of the rocket (plus engine) and find its weight. To do so we must convert the mass in grams to weight (due to gravity) in newtons. Below is the formula for this conversion:

$$W = mg$$

The mass (m) is measured in kg and the g represents the gravitational constant measured in m/s^2. On the Earth the gravitational constant is $9.8 m/s^2$. Returning to our original forumla we can see that 1 is subtracted from the division of the Force into the Weight. This is to allow for the pull of gravity (1g).

Analyzing a Flight

In our lesson Flight Path of a Model Rocket, we measured the maximum velocity of the flight using an electronic altimeter. Using some of the data from that flight we may now determine what the maximum velocity that the rocket should have been using our formula above.

We know that we used a B6-4 motor for the flight and we have in figure 1 above the thrust curve for this motor. From the thrust curve we know that the average thrust of this motor is 5.2N. From the flight analysis we measured the burn time at 0.8s (this is more accurate than using the thrust curve for the burn time). What is left now is to determine the average weight of the rocket.

We measured the mass of the rocket before lift-off at 106g. Using motor specifications from the ThrustCurve.org website we find that the propellent mass of a B6 motor is 6g giving the rocket a mass of 100g after the burn. Taking an average of the two values gives us:

$Mass_{avg} = (106g + 100g) / 2 = 103g$

To determine the average weight of the rocket in newtons we must first convert the mass from grams to kilograms and multiple the result by the gravitational constant of $9.8 m/s^2$:

$Weight_{avg} = (103/1000) * 9.8 = 1.0094N$

Below is a table of all the values we need to determine the maximum velocity:

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We may now determine the what the theoretical maximum velocity of the flight:

$$velocity = (\frac{5.1N}{1.0094N} - 1) \times 9.8\frac{m}{s^2} \times 0.8\frac{m}{s} = 31.77\frac{m}{s}$$

Aerodynamic Drag

In the Flight Path of a Model Rocket lesson, we found that the maximum velocity measured was 23 m/s. As we can see this rate is slower than the calculated maximum velocity. Thus we can conclude that aerodynamic drag affected the velocity of the rocket by a difference of 8.8m/s. In our lesson Understanding Rocket Aerodynamics, we discussed the factors that create aerodynamic drag. If we were to apply some of these concepts we would certainly close the gap between the calculated maximum velocity and the actual recorded one.

Mathematics in Motion

As we can see rocketry may be used to demonstrate mathematic concepts. It may also be used to demonstrate the effects of aerodynamic drag on moving objects. A good experiment to try would be to use motors with differing average thrust values and compare the results. As well, sanding the fins to be more airfoil shape would be a good experiment to try.

Reference Documentation

The formula for velocity used in this article came from the document ‘2844_Estes_Math_of_Model_Rocketry_TN-5’ by Robert L Cannon.

Mathematics in Motion – How high did my rocket go?

Learning Objective	In this lesson the student will use mathematics to determine the altitude of a rocket flight. By the end of this lesson the student should have an appreciation for the real world use of mathematics as it relates to the flight of a model rocket.
Grade Level	9 – 12

Figure 1 - Inclinometer

How high did my rocket go? This is probably one of the most asked questions in rocketry. There are a few ways in which to determine a rocket's apogee (maximum height). One is through the use of electronic altimeters. Altimeters are made by a few manufacturers with varying price points and complexities and can accurately determine and display the altitude of the rocket's flight. Another way is to use simple trigonometry based on the angle you physically measure (using an inclinometer) and your distance from the launch pad. In this lesson we will describe using math to determine the altitude of your rocket flight.

The Inclinometer

If you were to stand next to a tall tree or building and look at the top of it you will notice that your head is pointing up at a certain angle relative to the ground. Using an inclinometer you are able to measure that angle simply by pointing it to the top of the object and observing the angle recorded on the tool. Not only are inclinometers relatively inexpensive, they can be made using common materials.

The key component of the inclinometer is the weight-based arm that stays perpendicular to the ground as the tool is raised at an angle. Figure 1 shows a common inclinometer developed specifically for model rocketry.

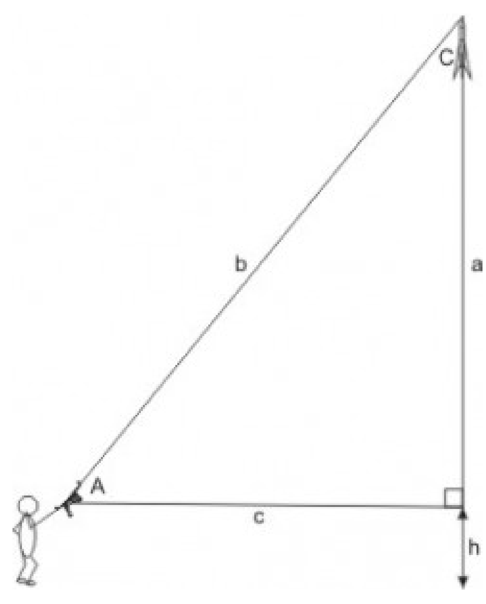

Figure 2 - Measuring rocket altititude

Forestry Usage

In forestry, inclinometers or clinometers as they are called in the field are used in measurements of tree height and terrain slope. A forester will stand at a fixed distance from the base of the tree and observe the top of the tree using the clinometer. More accurate results will be obtained if the height of the tree is less than the distance used for measurement thereby keeping the observed angle at less than 45 degrees.

Using this angle, the measured distance from the base of the tree, and simple trigonometry, the height of the tree can be determined. We can use this same method to determine the apogee of our model rocket flights.

Determining the Height of a Model Rocket

Imagine a triangle formed by the apogee of the rocket, the distance that the observer stands from the launch pad and the line connecting the observer and the apogee of the rocket (figure 2). We can measure the distance from the launch pad (c) as well as the angle that the observer takes when viewing the rocket with an inclinometer (A). We also know the height of the observer, or more specifically the distance to the ground from eye level for the observer (h). Using these variables we plug them into our simple trigonometric formula shown below.

$$a = c * tan(A) + h$$

So for example if the distance from the pad (c) was 150 metres, the angle measured (A) was 45 degrees and the eye level height of the observer is 1.6 metres, the altitude of the rocket would be:

*altitude = 150 * tan(45) + 1.6 OR altitude = 150 * 0.9999999999 + 1.6 = 151.59 metres*

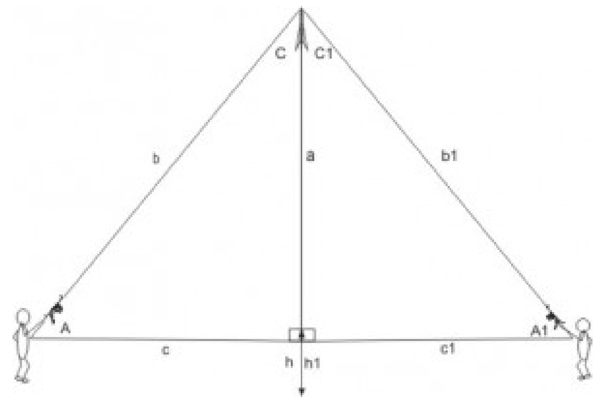

Figure 3 - Using two measurements to determine apogee

A More Accurate Method

Even a slight wind will cause most model rockets to weathercock, a term given to the flight of the model rocket into the wind. This causes great inaccuracies with the measurement. If the rocket were to fly towards the observer than the distance (c) used in the calculation would be greater than the actual distance. Thus the calculation for the altitude would produce a higher number than what it should be as the angle (A) would be based on a shorter distance to the launch pad rather than the distance measured. If the rocket were to weathercock away from the observer the opposite would be true.

A good way to reduce the error due to wind would be to have the observer stand so that he/she is perpendicular to the wind. Thus the distance (c) would be close to the actual distance as the rocket should veer left or right to the observer and not forward or back.

Another way would be to have multiple observers each taking measurements and averaging the results. You can see this method shown in figure 3.

As an example let's plug in some numbers. Observer 1 is 150 metres (c) from the pad, has a height of 1.6 metres (h) and measures an angle of 45 degrees (A). Observer 2 is 150 metres (c1) from the pad, has a height (eye level) of 1.3 meters (h1) and measures an angle of 35 degrees (A1). The altitude would then be the average of the two measurements as such:

*altitude recorded for Observer 1 = 150 * tan(45) + 1.6 = 151.59 metres*

*altitude recorded for Observer 2 = 150 * tan(35) + 1.3 = 106.33 metres*

altitude = (altitude recorded for Observer 1 + altitude recorded for Observer 2) / 2

OR

(151.59 + 106.33) / 2 = 132.96 metres.

The more observers you have taking the measurements the more accurate the result will be as the angle recorded by the inclinometer is very much dependent on the skills of the observer. Having a greater number of values to average out should result in a more accurate result. As well, you may want to avoid having someone measure the altitude of their own rocket. I have personally witnessed that rockets owned by the observer tend to go higher than the rest.

Mathematics in Motion

One of the greatest benefits in the study of model rocketry is the physical use of math and science skills. By applying simple mathematics to finding the solution for the apogee of a model rocket, it is easy to demonstrate a real world application of mathematics. As stated in our title, rocketry is mathematics in motion.